Homework #5 **Math 527, UNH fall 2011**
Due Friday Sept 30th in lecture

The usual homework rules apply: Always write equations, be organized and write legibly, use loose-leaf paper, staple in the upper-left corner, and write your name, section number, "Math 527", and "HW 5" in the upper-right corner. 1 point of 10 deducted for each.

Problem 1: Using Euler's formula $e^{ix} = \cos x + i \sin x$, show that $\cos x = (e^{ix} + e^{-ix})/2$ and that $\sin x = (e^{ix} - e^{-ix})/(2i)$.

Problem 2:
(a) Show that the general solution $y(t) = c_1 \cos \omega t + c_2 \sin \omega t$ of the mass-spring system $my'' + ky = 0$, where $\omega = \sqrt{k/m}$, can be written in the form $y(t) = A \sin(\omega t + \phi)$, where A is the *amplitude* and ϕ is the *phase angle*.
(b) What are A and ϕ in terms of c_1 and c_2?
(c) What are c_1 and c_2 in terms of A and ϕ?

Problem 3: For the damped harmonic oscillator or mass-spring-dashpot system $my'' + \beta y' + ky = 0$, what is the relation between m, β, and k that results in
(a) overdamped solutions,
(b) critically damped solutions, and
(c) underdamped solutions?
Express your answers as inequalities with β alone on the left-hand side.

Problems 4-6
(a) Use the method of judicious guessing to find a particular solution to the given ODE.
(b) Find the general solution by combining the particular solution with the general solution of the associated homogeneous problem.

Problem 4: $y'' + y' - 2y = 2x$

Problem 5: $y'' + 4y = x^2 + e^x$

Problem 6: $y'' + 4y = 3 \sin 2x$