Homework #12 **Math 527, UNH fall 2011**

Practice problems for exam #4, do not turn in. You should be able to do problems 2-4 without trouble. If you do have trouble, you should practice on more problems of this kind from Zill Exercises 8.2.

Problem 1: Suppose that the nth order linear system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ has a pair of complex eigenvalues $\lambda_1 = \mu + i\omega$ and $\lambda_2 = \mu - i\omega$ and eigenvectors $\mathbf{v}_1 = \mathbf{a} + i\mathbf{b}$ and $\mathbf{v}_2 = \mathbf{a} - i\mathbf{b}$. Use Euler's formula $e^{i\omega t} = \cos\omega t + i\sin\omega t$ to show that the solution

$$\mathbf{x}(t) = C_1\mathbf{v}_1 e^{\lambda_1 t} + C_2\mathbf{v}_2 e^{\lambda_2 t}$$

can be written in terms of sines and cosines as

$$\mathbf{x}(t) = c_1(\mathbf{a}\cos\omega t - \mathbf{b}\sin\omega t)e^{\mu t} + c_2(\mathbf{b}\cos\omega t + \mathbf{a}\sin\omega t)e^{\mu t}$$

for an appropriate choice of c_1, c_2 in terms of C_1, C_2. Note that if c_1, c_2 are real-valued, so is the latter expression for $\mathbf{x}(t)$.

Problem 2: Find the general solution of

$$\mathbf{x}' = \begin{pmatrix} -10 & -5 \\ 8 & 12 \end{pmatrix} \mathbf{x}$$

Problem 3: Express the general solution in terms of both complex exponentials and sines and cosines.

$$\mathbf{x}' = \begin{pmatrix} 6 & 1 \\ -5 & 4 \end{pmatrix} \mathbf{x}$$

Problem 4: Find the general solution of

$$\mathbf{x}' = \begin{pmatrix} 2 & 4 & 4 \\ -1 & -2 & 0 \\ -1 & 0 & -2 \end{pmatrix} \mathbf{x}$$

Problem 5: Find the general solution using a power series expansion about $x = 0$

$$(1 - x)y'' + y = 0$$

Problem 6: Find the general solution using a power series expansion about $x = 0$

$$y'' + x^2 y = 0$$