Solutions (or at least answers) to the practice final exam.

1

Solve homogeneous problem using ansatz: $y(x) = e^{\lambda x}$.

$$\lambda = 3 \Rightarrow y_h = c_1 e^{3x}$$

Use judicious guessing for y_p: $y_p = Ax + B$. Plug this guess in and solve for A and B.

$$A = -\frac{1}{3}; \quad B = -\frac{1}{9}$$

$$y = c_1 e^{3x} - \frac{1}{3}x - \frac{1}{9}$$

2

Solve homogeneous problem as in first problem:

$$\lambda = \pm 2 \Rightarrow y_h = c_1 e^{2x} + c_2 e^{-2x}$$

Use judicious guessing: $y_p = Axe^{2x}$. Note that we have multiplied our initial thought for a guess by x, because our homogeneous solution has the same form as the right hand side. Plug in and solve for A.

$$A = 3$$

$$y = c_1 e^{2x} + c_2 e^{-2x} + 3xe^{2x}$$

3

Find the homogeneous solutions y_1 and y_2 using the same ansatz: $y(x) = e^{\lambda x}$.

$$\lambda = \pm 1$$
$$y_1 = e^x$$
$$y_2 = e^{-x}$$

Now use variation of parameters to find $y_p = u_1 y_1 + u_2 y_2$. Use the formulas you should have to find u_1 and u_2.

$$W(y_1, y_2) = -2$$

$$u_1 = \frac{1}{4}(e^x + \frac{1}{3}e^{-3x})$$
$$u_2 = -\frac{1}{4}(\frac{1}{3}e^{3x} + e^{-x})$$

Thus,

$$y = c_1 e^x + c_2 e^{-x} + \frac{1}{4}(e^{2x} + \frac{1}{3}e^{-2x}) - \frac{1}{4}(\frac{1}{3}e^{2x} + e^{-2x})$$
$$= c_1 e^x + c_2 e^{-x} + \frac{1}{3}\sinh 2x$$

4

Solution should be

$$y(t) = \mathcal{U}(t - 2\pi)(\frac{1}{3}\sin t - \frac{1}{6}\sin 2t) + \cos 2t$$

5

The eigenvalues of $\mathbf{A}$ are: $\lambda_1 = 2$ and $\lambda_\pm = 5 \pm \sqrt{7}$. The corresponding eigenvectors are $\mathbf{v_1} = [1, 0, 1]$ and $\mathbf{v}_\pm = [-6, -1 \mp \sqrt{7}, -2]$.
Thus the complex-valued solution is:

$$y(t) = c_1\mathbf{v_1}e^{2t} + c_2\mathbf{v}_+e^{(5+\sqrt{7})t} + c_3\mathbf{v}_-e^{(5-\sqrt{7})t}$$

And the real-valued solution is:

$$y(t) = \tilde{c}_1\mathbf{v_1}e^{2t}e^{5t}\left[c_1\left(\mathbf{a}\cos(\sqrt{7}t) - \mathbf{b}\sin(\sqrt{7}t)\right) + \tilde{c}_2\left(\mathbf{b}\cos(\sqrt{7}t) + \mathbf{a}\sin(\sqrt{7}t)\right)\right]$$

where $\mathbf{a} = [-6, -1, -2]$ and $\mathbf{b} = [0, -\sqrt{7}, 0]$.

6

See previous problem sets etc.