HW #1 Solutions

1. $\frac{d}{dx} 6x^3 = 18x^2$

2. $\frac{d}{dx} 2x^{-1} = -2x^{-2} = \frac{-2}{x^2}$

3. $\frac{d}{dx} ax^n = nax^{n-1}$

4. $\frac{d}{dx} \sum_{n=0}^{N} a_n x^n = \sum_{n=1}^{N} n \cdot a_n x^{n-1}$

5. $\frac{d}{dt}(a \,\mathrm{Cos}\, wt + b \,\mathrm{Sin}\, wt) = a(-\mathrm{Sin}\, wt)(w) + b(\mathrm{Cos}\, wt)($

$= -aw \,\mathrm{Sin}\, wt + bw \,\mathrm{Cos}\, wt$

6. $\frac{d}{dx} e^{\alpha x} = e^{\alpha x} \cdot \alpha = \alpha e^{\alpha x}$

7. $\frac{d}{dx} \ln \mu x = \frac{1}{\mu x} \cdot \mu = \frac{1}{x}$

8. $\frac{d}{dx} \mathrm{Sin}\, \alpha x^2 = (\mathrm{Cos}(\alpha x^2)) \cdot (2\alpha x) =$

$= 2\alpha x \,\mathrm{Cos}\, \alpha x^2$

10. $\frac{d}{dx} \frac{x^2}{\sin \alpha x} = \frac{2x \sin \alpha x - x^2 \cos \alpha x \, (\alpha)}{(\sin \alpha x)^2} =$

$$= \frac{2x \sin \alpha x - \alpha x^2 \cos \alpha x}{(\sin \alpha x)^2}$$

11. $\frac{d}{dx} \sum_{n=0}^{\infty} \frac{1}{n!} \lambda^n x^n = \sum_{n=1}^{\infty} \frac{1}{n!} \lambda^n \cdot n x^{n-1} =$

$$= \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \lambda^n x^{n-1}$$

12. $\frac{d}{dx} \int f(x)\, dx = \frac{d}{dx} \Big(F(x) + C \Big) = f(x) + 0 = f($

13. $\frac{d}{dx} \int_0^x f(s)\, ds = \frac{d}{dx} \left(F(s) \Big|_0^x \right) = \frac{d}{dx} \Big(F(x) - F(0)$

$$= f(x) - 0 = f(x)$$

14. $\int 8x^3 dx = \frac{8x^4}{4} + C = 2x^4 + C$

15. $\int^1 8x^3 \ldots = 2x^4\Big|^1$

16. $\int_0^y 8x^3\,dx = 2x^4\Big|_0^y = 2y^4 - 0 = 2y^4$

17. $\int \sum_{n=0}^{N} a_n x^n\,dx = \sum_{n=0}^{N} a_n \frac{x^{n+1}}{n+1} + C$

18. $\int \frac{1}{x}\,dx = \ln|x| + C$

19. $\int \frac{d}{dx} f(x)\,dx = f(x) + C$

20. $\int \frac{dy}{dx}\,dx = \int \frac{d}{dx} y(x)\,dx = y(x) + C$

21. $\int \frac{d^n y}{dx^n}\,dx = \frac{d^{n-1} y}{dx^{n-1}} + C$

22. $\int y\,dx = \int y\,dx$

23. $\int \ln x\,dx$ let $u = \ln x$ $dv = dx$

$du = \frac{1}{x}\,dx$ $v = x$

24.

$$\int \tan^{-1} x \, dx =$$

let $u = \tan^{-1} x \qquad dv = dx$

$du = \frac{1}{1+x^2}\,dx \qquad v = x$

By Int. by parts, $\int \tan^{-1} x \, dx = x \tan^{-1} x - \int \frac{x}{1+x^2}\,dx$

Now, let $u = 1 + x^2$

$du = 2x\,dx \quad ; \quad dx = \frac{du}{2x}$

Then, $\int \frac{x}{1+x^2}\,dx = \frac{1}{2} \int \frac{2x}{1+x^2}\,dx = \frac{1}{2} \int \frac{1}{u}\,du =$

$= \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln |1 + x^2| + C$

Thus, $\int \tan^{-1} x \cdot dx = x \tan^{-1} x - \frac{1}{2} \ln |1+x^2| + C$

25. $\int \sum_{n=0}^{\infty} \frac{1}{n!} \lambda^n x^n \, dx = \sum_{n=0}^{\infty} \frac{1}{n!} \lambda^n \frac{x^{n+1}}{n+1} =$

$$= \sum_{n=0}^{\infty} \frac{1}{(n+1)!} \lambda^n x^{n+1}.$$

26.

$$3x^2 - 2y = 0 \quad , \quad 4x + y = 1$$

i) Solve one eq for one variable:

$$4x + y = 1 \Rightarrow y = 1 - 4x$$

ii) Substitute:

$$3x^2 - 2(1-4x) = 0$$

$$3x^2 - 2 + 8x = 0$$

iii) Solve for other variable:

Apply Quad. Form. to $3x^2 + 8x - 2 = 0$ with $a = 3$, $b = 8$, and $c = -2$

Thus, $x = \dfrac{-8 \pm \sqrt{64 - 4(3)(-2)}}{6} =$

$$= \frac{-8 \pm \sqrt{88}}{6} = \frac{-4 \pm \sqrt{22}}{3}$$

iv) Find corresponding y values:

For $x = \dfrac{-4 + \sqrt{22}}{3}$, $y = 1 - 4\left(\dfrac{-4 + \sqrt{22}}{3}\right)$

For $x = \dfrac{-4 - \sqrt{22}}{}$, $y = 1 - 4\left(\dfrac{-4 - \sqrt{22}}{}\right)$