

HW #1 Key

① $18x^2$

② $-2x^{-2}$

③ anx^{n-1}

④ $\sum_{n=1}^{\infty} nanx^{n-1}$

⑤ $-aw \sin wt + bw \cos wt$

⑥ $\alpha e^{\alpha x}$

⑦ $\frac{1}{\mu x} - \mu = \frac{1}{x}$

⑧ $\cos \alpha x^2 (2\alpha x)$

⑨ $2x \sin \alpha x + x^2 \alpha \cos \alpha x$

⑩ $\frac{2x \sin \alpha x - x^2 \alpha \cos \alpha x}{(\sin \alpha x)^2}$

⑪ $\sum_{n=1}^{\infty} \frac{1}{n!} \lambda^n - n x^{n-1} =$

$= \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \lambda^n x^{n-1}$

⑫ $\frac{d}{dx} \int f(x) dx =$

$\frac{d}{dx} [F(x) + C] = f(x)$

⑬ $\frac{d}{dx} \int_0^x f(s) ds = \frac{d}{dx} [F(x) - F(0)] = f(x)$

⑭ $2x^4 + C$

⑮ $\int_0^1 8x^3 dx = 2x^4 \Big|_0^1 = 2 - 0 = 2$

⑯ $\int_0^4 8x^3 dx = 2x^4 \Big|_0^4 = 2 \cdot 4^4$

⑰ $\sum_{n=0}^{\infty} a_n \frac{x^{n+1}}{n+1} + C$

⑱ $\ln|x| + C$

⑲ $f(x) + C$

$$\textcircled{20} \int \frac{dy}{dx} dx = y(x) + C$$

$$\textcircled{21} \int \frac{d^ny}{dx^n} dx = \frac{d^{n-1}y}{dx^{n-1}} + C$$

$$\textcircled{22} \int y dx = \int y dx$$

$$\textcircled{23} \int \ln x dx$$

$$\begin{aligned} \text{let } u &= \ln x & dv &= 1 dx \\ du &= \frac{1}{x} dx & v &= x \end{aligned}$$

I. B. P. yields

$$\int \ln x dx = x \ln x - \int x \frac{1}{x} dx$$

$$= x \ln x - \int dx = x \ln x - x + C$$

$$\textcircled{24} \int \tan^{-1} x dx = \int \text{Arctan } x dx$$

$$\begin{aligned} \text{let } u &= \text{Arctan } x & dv &= dx \\ du &= \frac{1}{x^2+1} & v &= x \end{aligned}$$

$$\int \text{Arctan } x dx = x \text{Arctan } x - \int \frac{x}{x^2+1} dx =$$

$$= \text{Arctan } x - \frac{1}{2} \int \frac{1}{u} du = \text{Arctan } x - \frac{1}{2} \ln |u| + C$$

$$= \text{Arctan } x - \frac{1}{2} \ln |x^2+1| + C$$

$$\textcircled{25} \int \sum_{n=0}^{\infty} \frac{1}{n!} \lambda^n x^n dx =$$

$$\sum_{n=0}^{\infty} \frac{1}{n!} \lambda^n \frac{x^{n+1}}{n+1} =$$

$$= \sum_{n=0}^{\infty} \frac{1}{(n+1)!} \lambda^n x^{n+1}$$

$$\textcircled{26} 3x^2 - 2y = 0$$

$$4x + y = 1$$

$$\Rightarrow y = 1 - 4x$$

$$3x^2 - 2(1 - 4x) = 0$$

$$3x^2 - 2 + 8x = 0$$

$$x = \frac{-4 \pm \sqrt{22}}{3} \quad \text{or}$$

$$\left(\frac{-4 + \sqrt{22}}{3}, 1 - 4\left(\frac{-4 + \sqrt{22}}{3}\right) \right)$$

$$\left(\frac{-4 - \sqrt{22}}{3}, 1 - 4\left(\frac{-4 - \sqrt{22}}{3}\right) \right)$$