problem # 1

Math 527
HW # 11
Solution Set

$$2x - y = 8$$
$$6x - 5y = 32$$

a)

$$\begin{pmatrix} 2 & -1 \\ 6 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 32 \end{pmatrix}$$

b) $\text{Det } A = \begin{vmatrix} 2 & -1 \\ 6 & -5 \end{vmatrix} = (2)(-5) - (6)(-1) =$

$$= -10 + 6 = -4 \neq 0$$

c) One.

d)

$$\left(\begin{array}{cc|c} 2 & -1 & 8 \\ 6 & -5 & 32 \end{array}\right) \xrightarrow{\frac{1}{2} R_1} \left(\begin{array}{cc|c} 1 & -\frac{1}{2} & 4 \\ 6 & -5 & 32 \end{array}\right) \xrightarrow{-6R_1 + R_2}$$

$$\left(\begin{array}{cc|c} 1 & -\frac{1}{2} & 4 \\ 0 & -2 & 8 \end{array}\right) \xrightarrow{-\frac{1}{2} R_2} \left(\begin{array}{cc|c} 1 & -\frac{1}{2} & 4 \\ 0 & 1 & -4 \end{array}\right) \xrightarrow{\frac{1}{2} R_2 + R_1}$$

$$\left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -4 \end{array}\right) \Rightarrow x = 2, \; y = -4$$

$$\underline{X} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

e) One : $\underline{X} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

problem # 2

$y + z = 6$

$3x - y + z = -7$

$x + y - 3z = -13$

a) $\begin{pmatrix} 0 & 1 & 1 \\ 3 & -1 & 1 \\ 1 & 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ -7 \\ -13 \end{pmatrix}$

b) $\det A = \begin{vmatrix} 0 & 1 & 1 \\ 3 & -1 & 1 \\ 1 & 1 & -3 \end{vmatrix} =$

$= 0 \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} - (1) \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} + (1) \begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix} =$

$= (-1)\left[(3)(-3) - (1)(1)\right] + \left[(3)(1) - (1)(-1)\right] =$

$= -1(-9-1) + (3+1) = 10 + 4 = 14 \neq 0$

c) One.

d)

$$\left(\begin{array}{ccc|c} 0 & 1 & 1 & 6 \\ 3 & -1 & 1 & -7 \\ 1 & 1 & -3 & -13 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 1 & -3 & -13 \\ 3 & -1 & 1 & -7 \\ 0 & 1 & 1 & 6 \end{array}\right) \xrightarrow{-3R_1 + R_2}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -3 & -13 \\ 0 & -4 & 10 & 32 \\ 0 & 1 & 1 & 6 \end{array}\right) \xrightarrow{-\frac{1}{4} \cdot R_2} \left(\begin{array}{ccc|c} 1 & 1 & -3 & -13 \\ 0 & 1 & -\frac{5}{2} & -8 \\ 0 & 1 & 1 & 6 \end{array}\right) \xrightarrow{-R_2 + R_3}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -3 & -13 \\ 0 & 1 & -\frac{5}{2} & -8 \\ 0 & 0 & \frac{7}{2} & 14 \end{array}\right) \xrightarrow{\frac{2}{7} \cdot R_3} \left(\begin{array}{ccc|c} 1 & 1 & -3 & -13 \\ 0 & 1 & -\frac{5}{2} & -8 \\ 0 & 0 & 1 & 4 \end{array}\right) \xrightarrow{-R_2 + R_1}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & -5 \\ 0 & 1 & -\frac{5}{2} & -8 \\ 0 & 0 & 1 & 4 \end{array}\right) \xrightarrow{\frac{1}{2} R_3 + R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & -\frac{5}{2} & -8 \\ 0 & 0 & 1 & 4 \end{array}\right) \xrightarrow{\frac{5}{2} R_3 + R_2}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \end{array}\right) \Rightarrow \quad x = -3, \; y = 2, \; z = 4$$

$$\underline{x} = \begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$$

e) One : $\underline{x} = \begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$

Problem #3

$$x_1 + 4x_2 - 2x_3 = 2$$
$$2x_1 + 7x_2 - x_3 = -2$$
$$2x_1 + 9x_2 - 7x_3 = 1$$

a)

$$\begin{pmatrix} 1 & 4 & -2 \\ 2 & 7 & -1 \\ 2 & 9 & -7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

b)

$$\det A = (1)\begin{vmatrix} 7 & -1 \\ 9 & -7 \end{vmatrix} - (4)\begin{vmatrix} 2 & -1 \\ 2 & -7 \end{vmatrix} - (2)\begin{vmatrix} 2 & 7 \\ 2 & 9 \end{vmatrix} =$$

$$= [(7)(-7) - (9)(-1)] - 4[(2)(-7) - (2)(-1)]$$
$$- 2[(2)(9) - (2)(7)] =$$

$$= -49 + 9 - 4(-14 + 2) - 2(18 - 14) =$$

$$= -40 + 48 - 8 = 0$$

c) Zero or inf. many

d)

$$\left(\begin{array}{ccc|c} 1 & 4 & -2 & 2 \\ 2 & 7 & -1 & -2 \\ 2 & 9 & -7 & 1 \end{array}\right) \xrightarrow{-2R_1 + R_2} \left(\begin{array}{ccc|c} 1 & 4 & -2 & 2 \\ 0 & -1 & 3 & -6 \\ 2 & 9 & -7 & 1 \end{array}\right) \xrightarrow{-2R_1 + R_3}$$

$$\left(\begin{array}{ccc|c} 1 & 4 & -2 & 2 \\ 0 & -1 & 3 & -6 \\ 0 & 1 & -3 & -3 \end{array}\right) \xrightarrow{-1 \cdot R_2} \left(\begin{array}{ccc|c} 1 & 4 & -2 & 2 \\ 0 & 1 & -3 & 6 \\ 0 & 1 & -3 & -3 \end{array}\right) \xrightarrow{-1 \cdot R_2 + R_3}$$

$$\left(\begin{array}{ccc|c} 1 & 4 & -2 & 2 \\ 0 & 1 & -3 & 6 \\ 0 & 0 & 0 & -9 \end{array}\right) \Rightarrow \text{Then } 0 = -9 \ \text{※}$$

This system has no solution

Problem # 4

$$x_1 + 4x_2 - 2x_3 = 2$$

$$2x_1 + 7x_2 - x_3 = -2$$

$$2x_1 + 9x_2 - 7x_3 = 10$$

a)

$$\begin{pmatrix} 1 & 4 & -2 \\ 2 & 7 & -1 \\ 2 & 9 & -7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 10 \end{pmatrix}$$

b) $\det A = (1) \begin{vmatrix} 7 & -1 \\ 9 & -7 \end{vmatrix} - (4) \begin{vmatrix} 2 & -1 \\ 2 & -7 \end{vmatrix} - (2) \begin{vmatrix} 2 & 7 \\ 2 & 9 \end{vmatrix} =$

$$= [(7)(-7) - (9)(-1)] - 4[(2)(-7) - (2)(-1)]$$

$$- 2[(2)(9) - (2)(7)] =$$

$$= (-49 + 9) - 4(-14 + 2) - 2(18 - 14) =$$

$$= -40 - 4(-12) - 2(4) = -40 + 48 - 8 = 0$$

c) Zero or infinitely many

d)

$$\left(\begin{array}{ccc|c} 1 & 4 & -2 & 2 \\ 2 & 7 & -1 & -2 \\ 2 & 9 & -7 & 10 \end{array}\right) \xrightarrow{-2R_1 + R_2} \left(\begin{array}{ccc|c} 1 & 4 & -2 & 2 \\ 0 & -1 & 3 & -6 \\ 2 & 9 & -7 & 10 \end{array}\right) \xrightarrow{-2R_1 + R_3}$$

$$\left(\begin{array}{ccc|c} 1 & 4 & -2 & 2 \\ 0 & -1 & 3 & -6 \\ 0 & 1 & -3 & 6 \end{array}\right) \xrightarrow{-1 \cdot R_2} \left(\begin{array}{ccc|c} 1 & 4 & -2 & 2 \\ 0 & 1 & -3 & 6 \\ 0 & 1 & -3 & 6 \end{array}\right) \xrightarrow{-1 \cdot R_2 + R_3}$$

$$\left(\begin{array}{ccc|c} 1 & 4 & -2 & 2 \\ 0 & 1 & -3 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow[-4R_2 + R_1]{} \left(\begin{array}{ccc|c} 1 & 0 & 10 & -22 \\ 0 & 1 & -3 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Then,

$$X_1 + 10X_3 = -22$$

$$X_2 - 3X_3 = 6$$

Solve for X_3:

$$X_3 = \frac{-22 - X_1}{10}, \quad X_3 = \frac{X_2 - 6}{3}$$

Then,

$$X_1 = \frac{-10X_2 - 6}{3}$$

Thus,

$$\underline{X} = \begin{pmatrix} \frac{-10X_2 - 6}{3} \\ X_2 \\ \frac{X_2 - 6}{3} \end{pmatrix}$$

e) System has infinitely many solutions.