Homework #4 **IAM 961, UNH fall 2011**
Due Monday, Dec 12th

1. Write a function for reduction of a matrix A to Hessenberg form, $H = Q^*AQ$. The function should both H and Q. Test your function by applying it to a symmetric matrix (it should return a diagonal H), by checking that Q is unitary, that $\|H - Q^*AQ\|$ is small, and by comparing the eigenvalues of H to the eigenvalues of A (they should be the same).

2. Write a function that computes an eigenvalue, eigenvector pair of a symmetric matrix A via Rayleigh quotient iteration, given an initial guess for the eigenvector. Test the function with a random symmetric matrix A reduced to Hessenberg form H and a random initial guess for the eigenvector, and plotting $\|v^{(k+1)} - (\pm q_J)\|$ and $|\lambda^{(k)} - \lambda_J|$ versus k on a log-linear plot (log errors on vertical, linear k on horizontal), where q_J and λ_J are the eigenvector and eigenvalue of H that the algorithm ultimately converges on (following Trefethen's notation in chapter 27).

Do the convergence rates match the cubic convergence predicted by Theorem 27.3?

3 (a) Implement the Arnoldi iteration algorithm to estimate the eigenvalues of a matrix A iteratively. Test it against a random matrix A with eigenvalues distributed over the unit circle, plotting the error versus iteration number k of the first few Arnoldi estimates compared to the first few leading eigenvalues of A.

(b) Now set a few diagonal elements of A to have magnitude $O(2)$ to $O(10)$ and recompute. What changes with the convergence rate? Why?

4. SVD problem still in production.