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Introduction

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Prospectus: towards the development of high-fidelity models of wall turbulence at large Reynolds number

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Recent and on-going advances in mathematical methods and analysis techniques, coupled with the experimental and computational capacity to capture detailed flow structure at increasingly large Reynolds numbers, afford an unprecedented opportunity to develop realistic models of high Reynolds number turbulent wall-flow dynamics. A distinctive attribute of this new generation of models is their grounding in the Navier–Stokes equations. By adhering to this challenging constraint, high-fidelity models ultimately can be developed that not only predict flow properties at high Reynolds numbers, but that possess a mathematical structure that faithfully captures the underlying flow physics. These first-principles models are needed, for example, to reliably manipulate flow behaviours at extreme Reynolds numbers. This theme issue of *Philosophical Transactions of the Royal Society A* provides a selection of contributions from the community of researchers who are working towards the development of such models. Broadly speaking, the research topics represented herein report on dynamical structure, mechanisms and transport; scale interactions and self-similarity; model reductions that restrict nonlinear interactions; and modern asymptotic theories. In this prospectus, the challenges associated with modelling turbulent wall-flows at large Reynolds numbers are briefly outlined, and the connections between the contributing papers are highlighted.

This article is part of the themed issue 'Toward the development of high-fidelity models of wall turbulence at large Reynolds number'.

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1. Introduction

Research pertaining to the instantaneous dynamics, statistical structure and Reynolds number scaling behaviours of turbulent wall-flows has experienced a resurgence over the past two decades (e.g. [1–3]). This renaissance is largely attributable to the ever-accelerating pace of new findings being enabled by on-going advances in experimental measurements and numerical simulation. The result has been an improved understanding, expressed both in the conceptualization and mathematical representation, of the underlying flow physics. In concert with the novel application of analytical approaches, this understanding is being leveraged in the construction of a new class of predictive models of turbulent boundary layer dynamics at large Reynolds numbers. A distinctive trait of these new models is that, relative to the previous generation of wall-flow models, they increasingly and intentionally retain firmer grounding in the Navier–Stokes equations. The central aims of this theme issue of *Philosophical Transactions of the Royal Society A* are to document recent progress in developing physics-rich models of high Reynolds number boundary layers, and to describe the rapidly advancing areas of research pertinent to the construction of Navier–Stokes based models.

Some of the mathematical techniques described herein have been developed over many years. Nevertheless, as evidenced by recent workshops and symposia, their adaptation to high Reynolds number turbulent wall-flows has garnered considerable attention and gained increased focus over the past decade. An important catalyst for this focus was the wall-flow component of the *Nature of High Reynolds Number Turbulence* workshop held at the Isaac Newton Institute (INI) at Cambridge University in 2008. This workshop brought together experimentalists, working specifically on the problem of high Reynolds number turbulent wall-flows, with a leading cadre of modellers and applied mathematicians who were developing new analyses of transitional flow phenomena and the origins of pattern formation, and refining tools for the identification and characterization of *exact coherent structures* in transitional and turbulent wall-flows. Since the INI workshop, the two *Turbulent Wall-Flows* workshops hosted by the Integrated Applied Mathematics Program at the University of New Hampshire (in 2013 and 2015), the engineering flows component of the *Mathematics of Turbulence* workshop at the Institute for Pure and Applied Mathematics at the University of California Los Angeles (in 2014), and the transition and turbulence component of the *Recurrent Flows: the Clockwork Behind Turbulence* workshop at the Kavli Institute for Theoretical Physics at the University of California Santa Barbara (in 2017) have served to further these research thrusts. Concomitantly, the present theme issue documents important aspects of this research community's progress towards the development of high-fidelity models of high Reynolds number wall-flows.

(a) Significance of addressing the high Reynolds number challenge

The largest eddies within a wall-bounded turbulent flow have a size characteristic of the overall width of the flow, e.g. of the order of the pipe diameter. Conversely, the smallest scales of motion are determined by the dynamics in the immediate vicinity of the surface, where the turbulence interacts directly with the surface to transfer mass, momentum and heat. Accordingly, the size of the smallest eddy is estimated by forming a length scale using the mean wall shear stress and the kinematic viscosity of the fluid. The Reynolds number of the flow is approximately proportional to the ratio of these largest and smallest lengths, and thus with increasingly high Reynolds number it is apparent that the scales of motion in a turbulent wall-flow span an increasingly vast spectral range.

The dynamical equations for wall turbulence (and turbulence in general) are known, and have been for many years. Thus, the state of research regarding wall turbulence is sometimes described as *mature*, with the connotation often being that the associated research is incremental with diminished opportunity for innovation. Recent advances in experiments and simulation have, however, created a situation that defies this characterization, with the rate of important new findings now coming with impressive regularity [1,2]. The research described herein adds to and leverages these findings by documenting recent advances in theory and analysis that underpin an emergent suite of new models and modelling concepts that seek to overcome the intrinsic challenges of predicting high Reynolds number wall-flows.

An imperative to develop the means to predict high Reynolds number wall-flow stems from the fact that these flows have pervasive and long-term scientific and technological importance. Notably, even small modifications to engineering designs, or advances in predictive capabilities, can have lasting impact. For example, a reliable turbulent drag reduction of only a few per cent amounts to enormous annual fuel cost savings for both the commercial airline and shipping industries. Successful flow manipulations would also reduce pollutant emissions, which for commercial aircraft are increasingly likely to be regulated by governmental standards similar to those currently established for automobiles. High-fidelity models are required to ensure the desired reliability of such predictions, while the intrinsic *scale separation* phenomenon noted above requires that these models scale favourably with increasing Reynolds number.

(b) Importance of Navier–Stokes based models

The reduced models of present interest constitute an important innovation because they target a first-principles foundation that ensures sustained and reliable advances in the understanding, prediction and control of wall-flow turbulence. Engineering analyses upon which public safety can be entrusted are uniquely distinguished by their direct connection to the fundamental physical laws. Specifically, without a first-principles basis, engineering designs or predictive models are often relegated to a ‘build and test’ paradigm, with outcomes that are inherently limited to the range of empirically verified observations. Extrapolations beyond this range have dubious validity.

Over the past two decades, advances in computing power and experimental diagnostics have greatly expanded our capacity to generate accurate realizations of turbulent wall-flows. With future advances justifiably anticipated, it is crucial to consider how these advances are to be optimally leveraged. Numerical realizations (i.e. direct numerical simulations, DNS) require the integration of the instantaneous Navier–Stokes equations, while realizations from a physical experiment require sampling from an actual turbulent wall-flow. It should be noted that: (i) DNS realizations and most controlled experiments will remain limited to moderate parameter ranges for decades to come and (ii) both types of realizations generate vast datasets that increase in size like the Reynolds number raised to the $\approx \frac{9}{4}$ power. These points highlight at least two reasons why first-principles based reduced models provide a means for sustaining the added scientific value derived from these realizations. Namely, such models allow observations from low and moderate Reynolds number flows to be reliably connected to those at high Reynolds number, and these models inherently provide simplified representations of the dominant dynamical mechanisms as a function of Reynolds number. Again, a central point here is that (to within our present state of understanding) these aims can only be accomplished within frameworks that have their scientific integrity ensured through a first-principles basis.

One particular implication of the second point should be emphasized in relation to our continued capacity to derive proportionate levels of new understanding from ever-larger datasets. That is, the increasing size of modern datasets is rapidly out-pacing our ability to organize, understand and usefully distill their information content into comprehensible knowledge-level prediction and design tools. Without the development of high-fidelity reduced models, the investments devoted to producing these ever-larger datasets therefore may be expected to yield a proportionally diminishing return.

2. Scope of topics

Both the instantaneous and statistical properties of turbulent wall-flows derive from dynamical interactions across the scales of motion, as dictated by the Navier–Stokes equations. The nonlinearity of these equations and the range of dynamically active motions create considerable challenges for the construction of tractable yet well-founded representations of wall-flow dynamics. During the latter half of the past century, however, research began to reveal the spatially heterogeneous nature of transport in turbulent wall-flows. This complements and reinforces the realization that there are recurrent coherent motions within these flows, and that these motions carry with them the bulk of the dynamics. At first, research on coherent motions primarily focused on their identification and classification (e.g. [4,5]). More recent efforts have, however, sought to exploit and connect the dynamical significance of coherent motions in the construction of reduced models capable of capturing the essential dynamics in the high Reynolds number regime. Accordingly, the current issue presents results that can be loosely categorized into four research themes:

— dynamical structure, mechanisms and transport,
— scale interactions and self-similarity: observations and models,
— model reductions that selectively restrict nonlinear interactions, and
— asymptotically reduced models.

The contributions to these themes and their inter-connections are now briefly discussed.

(a) Dynamical structure, mechanisms and transport

As mentioned above, the recent progress towards the development of accurate models of high Reynolds number wall-flows is largely attributable to the complementary coalescence of detailed experimental observations (from both physical and numerical experiments) and the advancement of the mathematical tools associated with new modelling frameworks and analysis techniques. In this regard, experiments that continue to clarify the relationships between the spatial structure of the motions responsible for transport and the resulting dynamical structure of the flow continue to play a vital role. Four contributions herein nominally fall within this category.

Well-resolved quantifications of the statistical properties of wall turbulence, including accurate documentation of the variations of these properties with Reynolds number, constitute perhaps the most basic experimental need relative to model development and validation. For reasons associated with maintaining high spatial and temporal measurement resolution with increasing Reynolds number, there are distinct advantages to employing large-scale flow facilities that operate at relatively low speed (e.g. [6,7]). In this regard, the contribution by Örlü *et al.* [8] presents the first turbulent stress measurements in the newly constructed Long Pipe facility at the Center for International Cooperation in Long Pipe Experiments (CICLoPE) in Bologna, Italy. These detailed profile measurements are shown to largely reinforce previous results supporting Townsend's notion of attached eddies [9] and, more generally, the existence of an inertial sub-domain over which statistical profiles adhere to *distance-from-the-wall* scaling, a feature explored and leveraged by a number of studies in the present issue [10–12]. Relative to previous studies, these new CICLoPE results show some deviations in the Reynolds number dependencies of some of the statistics. They also seem to support the 'diagnostic' scaling proposed by Alfredsson *et al.* [13]—a finding that has potential implications regarding the invariance of the so-called Perry–Townsend constant (e.g. [14]).

Clear evidence has emerged over the past decade indicating the existence and importance of large-scale energetic motions, including their amplitude-modulating effect on the near-wall flow (e.g. [15,16]). Such observations have potential connection to recent analyses indicating the existence of *exact coherent structures* embedded in asymptotically high Reynolds number shear flows. The recent results of Deguchi & Hall [17,18] demonstrate that the Navier–Stokes equations admit important coherent states that are characterized by interactions that span the full width of the flow at asymptotically high Reynolds numbers. Their work provides a context that attaches broader relevance to the present contribution by Dogan *et al.* [19]. These authors investigate the influences of free stream turbulence on the large-scale outer motions, as well as the interaction of these motions with those near the wall. The results of Dogan *et al.* indicate that the large-scale motions characteristic of the inertial domain are not only energized by free stream turbulence, but also retain the same phase relationship with the small scale near-wall motions as observed in the canonical flow under increasing Reynolds number. These phase relationships are key to the so-called *inner–outer interactions*, which are also addressed by the present studies of Baars *et al.* [20] and Duvvuri & McKeon [21]. Overall, the findings by Dogan *et al.* suggest that free stream turbulence usefully produces structural features that mimic those of a higher Reynolds number condition.

A distinct advantage of DNS is its ability to capture three-dimensional flow structure and to accurately quantify difficult to measure quantities, such as those associated with the fluctuating pressure field. The contribution by Hellström & Smits [22] extends their previous Proper Orthogonal Decomposition (POD) based studies of the large-scale motions in turbulent pipe flow [23] to an investigation of the scales associated with the fluctuating pressure. Their analysis reveals convincing connections between the pressure signature of the large-scale pressure-bearing motions and those previously identified to have association with the velocity field. They also provide evidence that these motions exhibit self-similarity across an interior domain, as quantified by the *distance-from-the-wall* scaling exhibited by the POD modal peaks as well as by the scaled mode shapes themselves. These results reinforce the other results in this issue regarding the existence of self-similar statistical behaviours on the inertial sublayer [8,10–12], a result that is intriguing (and somewhat surprising) given the relatively low Reynolds number of the DNS dataset that Hellström and Smits employed.

Leverageable modelling strategies can be derived from the identification and representation of generic mechanisms. In this regard, the fundamental richness of the physics in Taylor–Couette flow and the flow in a Rayleigh–Bénard convection cell provide a natural setting for the study of such mechanisms. The contribution by Brauckmann *et al.* [24] herein uses two high resolution DNS of these flows to delve more deeply into the previously proposed analogy [25] between the momentum and heat transport mechanisms in Taylor–Couette and Rayleigh–Bénard flows, respectively. Brauckmann *et al.* find that, when the appropriately defined Nusselt number is used to guide comparisons, the transport properties of these flows exhibit remarkably similar behaviours in both their mean and fluctuating fields.

(b) Scale interactions and self-similarity: observations and models

Dimensional reasoning supports the aforementioned observation that the dynamically significant motions in wall-flows range in size from $O(\nu/u_\tau)$ to $O(\delta)$, where ν is the kinematic viscosity, u_τ is the friction velocity, and δ is the boundary layer thickness (or, e.g. channel half-height). As might naturally be expected, the scales of motion concentrate near $O(\nu/u_\tau)$ in the vicinity of the wall, and increase to $O(\delta)$ near the freestream or channel core. Consequently, it is quite natural to consider an interacting hierarchy of scales of motions, and the companion notion of distance-from-the-wall scaling. Indeed, recent analyses show that the mean equation of motion formally admits a continuous and self-similar hierarchy of scaling layers (e.g. [26]). Laboratory experiments over the past two decades provide evidence that collections of hairpin-like vortices form and evolve to generate larger scale coherent features [27], and models that incorporate a self-similar eddy hierarchy consonant with these observations have been studied and refined for decades (e.g. [9,28]). At least four contributions to the present issue investigate features associated with a hierarchy of scales of motion, as well as with interactions across these scales of motion.

Observations from field experiments at very high Reynolds number provide clear evidence that the motions in the immediate vicinity of the wall are energetically enhanced with increasing Reynolds number. This enhancement largely (but not wholly) stems from a growing low wavenumber (frequency) energy content of the larger scale motions away from the wall that is superposed upon the motions near the wall [29]. The part of the energy enhancement not associated with this superposition is, however, critical to understanding wall-flow dynamics, owing to its origin in the nonlinear interactions between scales. Within this context, the present contribution by Baars *et al.* [20] extends the recent findings of Marusic and co-workers showing that these nonlinear interactions between the large and small scales are associated with amplitude and frequency modulations (e.g. [16,30]). Here they provide a detailed presentation of the underlying spatial structure of the flow, and the associated Reynolds number dependencies. In so doing, they show that nonlinear modulation phenomena are also important to the outer region flow structure: a structure nominally comprised of uniform zones of streamwise momentum segregated by slender regions of intense vorticity [31]. The physics of these interactions connect to a number of studies herein [11,12,19], but perhaps most directly to the asymptotic theory of Chini *et al.* [32].

As discussed in both the studies by Dogan *et al.* [19] and Baars *et al.* [20], an essential ingredient of the interactions across scales is the existence of spatial phase relations among the motions involved. Within the context of modelling high Reynolds number wall flows, a broadly important question here pertains to whether the large- and small-scale motions, on average, asymptotically develop a well defined spatial phase relationship. More specific to the model frameworks advanced by McKeon and co-workers [11,33] and by Schmid & Sayadi [34], phase information provides important insights regarding how the nonlinear terms in their Navier–Stokes based models drive the linear response modes. In their present contribution, Duvvuri & McKeon [21] experimentally investigate the phase relationships between scales by precisely perturbing a turbulent boundary at two distinct frequencies via the use of an actuated wall motion. Their analysis elucidates mode interactions using an amplitude modulation correlation coefficient and triadic interactions associated with the sums and differences of wavenumbers. A longer term potential for their experimental methodology is to advance well-justified reduced models by clarifying the dominant underlying connectivity between scales.

An implicit premise of coherent motion research is that, amid the seemingly chaotic turbulent velocity and vorticity fields, there is an underlying order to the most significant dynamical processes. For obvious reasons, this concept is attractive relative to the development of reduced models of high Reynolds number flows. The contribution by Morrill-Winter *et al.* [10] explores evidence that the signature of this ordered structure withstands time averaging. The context for their study is established from analysis of the mean equations of motion (e.g. [26]). These analyses indicate that the motions responsible for wallward momentum transport are, on average, associated with a self-similar hierarchy of scaling layers that underlie a similarity solution to the mean momentum equation—resulting in a logarithmic mean velocity profile equation. Relative to the Reynolds shear-stress-producing motions, these analyses suggest that there exists a geometric structure that directly connects to the coordinate stretching parameter required to cast the mean equation in its self-similar form. Building upon the study of Klewicki *et al.* [35], Morrill-Winter *et al.* provide evidence that the geometry of the coordinate stretching is also reflected in the growth rate of the average streamwise length of the $-uv$ motions with distance from the wall, as well as in the value of the skewness of the $-uv$ fluctuations on the inertial sublayer. The latter of these suggests that there is a self-similar relation between the amplitude and spatial scale of the momentum transporting motions on this domain.

Model reductions suitable for predicting phenomena at high Reynolds number must not only retain the essential physics, but also attain the requisite computational efficiency. In the resolvent based model of McKeon & Sharma [33], efficiency is inherently gained by leveraging empirically derived knowledge of the (approximate) scales of the dominant flow structures. This information is used to represent the spatial structure of the set (typically small in number) of linear response modes that become most amplified by the nonlinear terms in the governing equations. The present contribution by Sharma *et al.* [11] further leverages an underlying self-similar geometric structure of these resolvent modes over an interior domain (inertial sublayer) that is self-consistently defined within their Navier–Stokes based framework. Analysis of the nonlinearly driven triadic interactions of these geometrically self-similar response modes suggests a means for compact representation of the flow structure on the inertial sublayer. Efficiency is attained by capturing the mode interactions at one wall-normal distance, and then rescaling these interactions to represent the physics at the other levels within the self-similar hierarchy. The self-similar structure they derive attains a level of consistency with attached-eddy modelling concepts [9] as well as with the structure admitted by the mean momentum equation (e.g. [10,26]). Physically, their model results show some intriguing similarities to the structure elucidated in the experiments of Baars *et al.* [20].

(c) Model reductions that selectively restrict nonlinear interactions

The complex richness of wall-flow dynamics largely stems from the nonlinear interactions of Navier–Stokes turbulence under the inhomogeneity imposed by a no-slip wall. Increasing the range of relevant scales expands this richness, and thus poses significant challenges for accurately predicting flow phenomena at high Reynolds number. Given such considerations, a number of important questions arise regarding which nonlinear interactions are essential to capture the dynamics, how these nonlinear interactions vary with distance from the wall, and similarly, whether these interactions evolve significantly with Reynolds number. The three studies described next address various aspects of these questions.

Restricted nonlinear models (RNL) of wall turbulence generically invoke some level of streamwise averaging, as they were: (i) at least partially motivated by the slowly varying streamwise structure of the roll-streak motions in near-wall turbulence and the associated self-sustaining (*exact coherent structure*) process model [36,37], and (ii) were partly justified by single streamwise-wavenumber systematic asymptotic reductions of the full Navier–Stokes equations [38–40]. Given a streamwise-averaged version of the Navier–Stokes equations (or more generally, a representation involving a small number of streamwise modes), equations for the fluctuating fields are developed, and model reduction is attained by neglecting the nonlinear terms involving fluctuation–fluctuation products (e.g. [41]). In their contribution, Farrell *et al.* [42] describe how the RNL ideas can be employed within a *statistical state dynamics* (SSD) framework that respectively represents the mean velocity and Reynolds stresses by the first and second cumulant of ensemble averages. These ensemble averages are constructed using a Leray projection of the Navier–Stokes equations and by employing temporal white noise forcing to generate the members of the ensemble. The solutions to the resulting equations for the mean and velocity covariances are investigated by invoking a closure condition on the second cumulant expansion—their so-called S3T model. The behaviour of the S3T model is then explored relative to different levels of streamwise modal truncation. Given its additional level of model reduction, the SSD approach holds considerable promise relative to capturing structural properties at high Reynolds numbers.

An emerging theme underlying a number of recent model constructions (such as RNL [42] and the resolvent model framework [11]) is that linear mechanisms play a greater role in the overall dynamics than perhaps previously imagined, and especially so with increasing Reynolds number. The contribution by Cossu & Hwang [12] summarizes their recent results, revealing elements that simultaneously incorporate a number of the physical and conceptual features studied by other authors in this theme issue. Here, they describe and provide evidence for the existence of a linear lift-up effect that incorporates streaks and rolls characteristic of the near-wall self-sustaining process, but that is generically replicated as a function of increasing scale with distance from the wall. In stark contrast to models in which the large-scale motions derive their structural characteristics from the concatenation of smaller scale motions (e.g. via spontaneous alignment of hairpin vortex packets), their findings suggest that a self-similar family of exact coherent structures support self-sustaining processes simultaneously throughout the boundary layer. This viewpoint rather naturally connects to Townsend's notion of an attached eddy hierarchy of motions, but in a manner that is physically distinct from most previous interpretations. Similarly, it is apparently consistent with the self-similar triadic interactions described by Sharma *et al.* [11], as well as with the self-similar hierarchy of scaling layers admitted by the mean dynamical equation [10]. Although developed from a different mathematical approach, the results of Cossu & Hwang also seem to be reinforced by the theory of Chini *et al.* [32], which similarly points to a self-sustaining process that derives from spatially localized autonomous mechanisms.

Another promising approach to determining the appropriate degree of nonlinearity to include within a model is to *query* the dynamics directly. In developing a low-dimensional model of near-wall region dynamics suitable for large eddy simulation, Schmid & Sayadi [34] effectively pursue such an approach. Here, they use the dynamic mode decomposition (DMD) (see Schmid [43]) to extract the dynamic mode contributions to the Reynolds shear stress. Their formulation employs a triple decomposition of the velocity field, consisting of mean, coherent and incoherent parts, and they extract the relevant DMD modes from DNS. In a manner having similarities to the resolvent-based modelling framework of McKeon & Sharma [33], this information is then fed into a Navier–Stokes based input–output model of the dynamics reflected in the first and second moments of the coherent structures. Like the resolvent model, a particularly attractive feature of the DMD based approach described by Schmid and Sayadi is its sparse matrix representation, leading to computational efficiency, and thus its suitability for high Reynolds number applications. Consistent with experimental observations and the near-wall exact coherent structures [36,37], their model properly recovers the highly streamwise-elongated spatial structure of the most dynamically significant near-wall motions.

(d) Asymptotically reduced models

Traditionally, asymptotic methods have provided a means for analytically describing statistical profiles in a way that inherently reflects the underlying Reynolds number scaling properties. Over the past two decades, however, asymptotic techniques have increasingly proven to be a powerful tool for generating reduced models (systems of equations) that have the additional benefit of formally representing the given physical system in extreme parameter regimes (e.g. [44]). Unlike flows that allow for an asymptotically simplified mathematical representation owing to the imposition of a strong external constraint [45], the high Reynolds number characteristics of turbulent wall-flows derive from the much more subtle condition of an increasing scale separation between the energetic and dissipative motions. In the final two contributions discussed in this prospectus, asymptotic descriptions of extreme Reynolds number flow structure are developed that inherently treat the relationships and interactions between these large- and small-scale motions.

It now seems apparent that the exact coherent structures (ECS) associated with the near-wall self-sustaining process (as described by Waleffe [36] and others) constitute a low Reynolds number version of the vortex–wave interaction (VWI) found via asymptotic analysis by Hall and co-workers (e.g. [46]). Over the last few decades, ECS research has shown how coherent structures in closed flows at moderate Reynolds result from *close passes* (in state space) of the turbulent dynamics to unstable invariant solutions of the Navier–Stokes equations [47,48]. This approach has considerable potential, as it provides a direct path from the Navier–Stokes equations to the dominant physical structures and dynamic organization of these flows via precise and finite numerical computations. The application of ECS formulations to wall flows at high Reynolds numbers requires, however, significant enhancements. These include the extension to open flows, computation of localized coherent structures in extended domains, understanding how localized structures interact, and the asympotics of these solutions at high Reynolds numbers. While inroads have recently been made in addressing localized solutions and open flows (e.g. [49–51]), Hall and co-workers have constructed asymptotic theories of ECS in open flows. Building upon these formulations, Deguchi & Hall [17] recently showed that an exact coherent solution distinct from that supported by a VWI is also present in the asymptotic suction boundary layer. This equilibrium solution captures nonlocal interactions between coherent motions that (essentially) reside in the freestream and the near-wall streaks. As such, their findings directly suggest an inner–outer interaction and, thus, have potential connection to observations in the experimental studies described herein by Dogan *et al.* [19] and Baars *et al.* [20]. In their present contribution, Deguchi & Hall [18] further elucidate the properties of these freestream coherent motions and how they interact with the near-wall streaks. Of particular note is their result that, under the proper conditions, relatively weak freestream disturbances can induce much larger amplitude streamwise vortices and streaks near the wall.

On the inertial domain, both observations and theoretical considerations indicate that the physical space realization of scale separation comes in the form of nearly uniform regions of streamwise momentum ('uniform momentum zones', UMZ) that are segregated by slender regions of significantly elevated vorticity ('vortical fissures', VF) [31,52]. The precise mechanisms that sustain these spatially adjacent motions, however, are far from clear. Guided by the magnitude ordering of terms emerging from analysis of the mean equations, Chini *et al.* [32] develop an asymptotically reduced, Navier–Stokes based theory that describes the interactions between a vortical fissure and the adjacent uniform momentum zones. Consistent with the perspectives of Cossu & Hwang [12], this model predicts that the coupled UMZ/VF structure is locally self-sustaining and, more specifically, that the driving turbulent stress divergence originates from a critical layer centred within the vortical fissure. Certain elements of the model are shared by VWI theory [38,46], but unlike the VWI scenario the contribution by Chini *et al.* incorporates an irreducible (inviscid) outer layer that respects the experimental observations of a high Reynolds number roll motion. These motions are responsible for homogenizing the streamwise momentum in the UMZ, and retain physical consistency with inertial layer dynamics.

3. Summary

As reflected in both the diversity and connectedness of the approaches described herein, there are now a number of promising avenues for developing efficient high-fidelity models for turbulent wall-flows at large Reynolds numbers. It is our hope that the contents of this theme issue serve to further promote this scientifically rich and rapidly advancing area of research. We are pleased to extend our thanks to all of the contributors to this theme issue and to the Royal Society for supporting its publication.

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References

1. Klewicki JC. 2010 Reynolds number dependence, scaling and dynamics of turbulent boundary layers. *J. Fluids Eng.* **132**, 0094001. (doi:10.1115/1.4002167)
2. Marusic I, McKeon BJ, Monkewitz PA, Nagib HM, Smits AJ, Sreenivasan KR. 2010 Wall-bounded turbulent flows at high Reynolds numbers: recent advances and key issues. *Phys. Fluids* **22**, 065103. (doi:10.1063/1.3453711)
3. Smits AJ, McKeon BJ, Marusic I. 2011 High-Reynolds number wall turbulence. *Annu. Rev. Fluid Mech.* **43**, 353–375. (doi:10.1146/annurev-fluid-122109-160753)
4. Cantwell BJ. 1981 Organized motion in turbulent flow. *Annu. Rev. Fluid Mech.* **13**, 457–515. (doi:10.1146/annurev.fl.13.010181.002325)
5. Robinson SJ. 1991 Coherent motions in the turbulent boundary layer. *Annu. Rev. Fluid Mech.* **23**, 601–639. (doi:10.1146/annurev.fl.23.010191.003125)
6. Hutchins N, Nickels TB, Marusic I, Chong MS. 2009 Hot-wire spatial resolution issues in wall-bounded turbulence. *J. Fluid Mech.* **635**, 103–136. (doi:10.1017/S0022112009007721)
7. Vincenti P, Klewicki JC, Morrill-Winter CT, White CM, Wosnik M. 2013 Streamwise velocity statistics in turbulent boundary layers that spatially develop to high Reynolds number. *Exp. Fluids* **54**, 1629. (doi:10.1007/s00348-013-1629-9)
8. Örlü R, Fiorini T, Segalini A, Bellani G, Talamelli A, Alfredsson PH. 2017 Reynolds stress scaling in pipe flow turbulence—first results from CICLoPE. *Phil. Trans. R. Soc. A* **375**, 20160187. (doi:10.1098/rsta.2016.0187)
9. Townsend AA 1976 *The structure of turbulent shear flow*, 2nd edn. Cambridge, UK: Cambridge University Press.

10. Morrill-Winter C, Philip J, Klewicki J. 2017 Statistical evidence of an asymptotic geometric structure to the momentum transporting motions in turbulent boundary layers. *Phil. Trans. R. Soc. A* **375**, 20160084. (doi:10.1098/rsta.2016.0084)
11. Sharma AS, Moarref R, McKeon BJ. 2017 Scaling and interaction of self-similar modes in models of high Reynolds number wall turbulence. *Phil. Trans. R. Soc. A* **375**, 20160089. (doi:10.1098/rsta.2016.0089)
12. Cossu C, Hwang Y. 2017 Self-sustaining processes at all scales in wall-bounded turbulent shear flows. *Phil. Trans. R. Soc. A* **375**, 20160088. (doi:10.1098/rsta.2016.0088)
13. Alfredsson PH, Örlü R, Segalini A. 2012 A new formulation for the streamwise turbulence intensity distribution in wall-bounded turbulent flows. *Eur. J. Mech. B* **36**, 167–175. (doi:10.1016/j.euromechflu.2012.03.015)
14. Örlü R, Segalini A, Klewicki J, Alfredsson PH. 2016 High-order generalization of the diagnostic scaling for turbulent boundary layers. *J. Turbulence* **17**, 664–677. (doi:10.1080/14685248.2016.1169282)
15. Hutchins N, Marusic I. 2007 Evidence of very long meandering features in the logarithmic region of turbulent boundary layers. *J. Fluid Mech.* **579**, 1–28. (doi:10.1017/S0022112006003946)
16. Mathis R, Hutchins N, Marusic I. 2009 Large-scale amplitude modulation of small-scale structures in turbulent boundary layers. *J. Fluid Mech.* **628**, 311–337. (doi:10.1017/S0022112009006946)
17. Deguchi K, Hall P. 2014 Free-stream coherent structures in parallel boundary-layer flows. *J. Fluid Mech.* **752**, 602–625. (doi:10.1017/jfm.2014.282)
18. Deguchi K, Hall P. 2017 The relationship between free-stream coherent structures and near-wall streaks at high Reynolds numbers. *Phil. Trans. R. Soc. A* **375**, 20160078. (doi:10.1098/rsta.2016.0078)
19. Dogan E, Hearst RJ, Ganapathisubramani B. 2017 Modelling high Reynolds number wall–turbulence interactions in laboratory experiments using large-scale free-stream turbulence. *Phil. Trans. R. Soc. A* **375**, 20160091. (doi:10.1098/rsta.2016.0091)
20. Baars WJ, Hutchins N, Marusic I. 2017 Reynolds number trend of hierarchies and scale interactions in turbulent boundary layers. *Phil. Trans. R. Soc. A* **375**, 20160077. (doi:10.1098/rsta.2016.0077)
21. Duvvuri S, McKeon B. 2017 Phase relations in a forced turbulent boundary layer: implications for modelling of high Reynolds number wall turbulence. *Phil. Trans. R. Soc. A* **375**, 20160080. (doi:10.1098/rsta.2016.0080)
22. Hellström LHO, Smits AJ. 2017 Structure identification in pipe flow using proper orthogonal decomposition. *Phil. Trans. R. Soc. A* **375**, 20160086. (doi:10.1098/rsta.2016.0086)
23. Hellström LHO, Smits AJ. 2014 The energetic motions in turbulent pipe flow. *Phys. Fluids* **26**, 125102. (doi:10.1063/1.4902436)
24. Brauckmann HJ, Eckhardt B, Schumacher J. 2017 Heat transport in Rayleigh–Bénard convection and angular momentum transport in Taylor–Couette flow: a comparative study. *Phil. Trans. R. Soc. A* **375**, 20160079. (doi:10.1098/rsta.2016.0079)
25. Eckhardt B, Grossmann S, Lohse D. 2007 Fluxes and energy dissipation in thermal convection and shear flows. *Europhys. Lett.* **78**, 24001. (doi:10.1209/0295-5075/78/24001)
26. Klewicki J. 2013 Self-similar mean dynamics in turbulent wall flows. *J. Fluid Mech.* **718**, 596–621. (doi:10.1017/jfm.2012.626)
27. Adrian RJ. 2007 Hairpin vortex organization in wall turbulence. *Phys. Fluids* **19**, 041301. (doi:10.1063/1.2717527)
28. Perry AE, Chong MS. 1982 On the mechanism of wall turbulence. *J. Fluid Mech.* **119**, 173–217. (doi:10.1017/S0022112082001311)
29. Metzger MM, Klewicki JC. 2001 A comparative study of near-wall turbulence in high and low Reynolds number boundary layers. *Phys. Fluids* **13**, 692–701. (doi:10.1063/1.1344894)
30. Ganipathisubramani B, Hutchins N, Monty J, Chong MS, Marusic I. 2012 Amplitude and frequency modulation in wall turbulence. *J. Fluid Mech.* **712**, 61–91. (doi:10.1017/jfm.2012.398)
31. Meinhart CD, Adrian RJ. 1995 On the existence of uniform momentum zones in a turbulent boundary layer. *Phys. Fluids* **7**, 694–696. (doi:10.1063/1.868594)
32. Chini GP, Montemuro B, White CM, Klewicki J. 2017 A self-sustaining process model of inertial layer dynamics in high Reynolds number turbulent wall flows. *Phil. Trans. R. Soc. A* **375**, 20160090. (doi:10.1098/rsta.2016.0090)

33. McKeon BJ, Sharma A. 2010 A critical layer framework for turbulent pipe flow. *J. Fluid Mech.* **658**, 336–382. (doi:10.1017/S002211201000176X)
34. Schmid PJ, Sayadi T. 2017 Low-dimensional representation of near-wall dynamics in shear flows, with implications to wall-models. *Phil. Trans. R. Soc. A* **375**, 20160082. (doi:10.1098/rsta.2016.0082)
35. Klewicki J, Philip J, Marusic I, Chauhan K, Morrill-Winter C. 2014 Self-similarity in the inertial region of wall turbulence. *Phys. Rev. E* **90**, 063015. (doi:10.1103/PhysRevE.90.063015)
36. Waleffe F. 1997 On a self-sustaining process in shear flows. *Phys. Fluids* **9**, 883–900. (doi:10.1063/1.869185)
37. Jiménez J, Pinelli A. 1999 The autonomous cycle of near-wall turbulence. *J. Fluid Mech.* **389**, 335–359. (doi:10.1017/S0022112099005066)
38. Hall P, Sherwin S. 2010 Streamwise vortices in shear flows: harbingers of transition and the skeleton of coherent structures. *J. Fluid Mech.* **661**, 178–205. (doi:10.1017/S0022112010002892)
39. Beaume C, Chini GP, Julien K, Knobloch E. 2015 Reduced description of exact coherent states in parallel shear flows. *Phys. Rev. E* **91**, 043010. (doi:10.1103/PhysRevE.91.043010)
40. Chini GP. 2016 Exact coherent structures at extreme Reynolds number. *J. Fluid Mech.* **794**, 1–4. (doi:10.1017/jfm.2016.154)
41. Marston JB, Chini GP, Tobias SM. 2016 Generalized quasilinear approximation: application to zonal jets. *Phys. Rev. Lett.* **116**, 214501. (doi:10.1103/PhysRevLett.116.214501)
42. Farrell BF, Gayme DF, Ioannou PJ. 2017 A statistical state dynamics approach to wall turbulence. *Phil. Trans. R. Soc. A* **375**, 20160081. (doi:10.1098/rsta.2016.0081)
43. Schmid P. 2010 Dynamic mode decomposition of numerical and experimental data. *J. Fluid Mech.* **656**, 5–28. (doi:10.1017/S0022112010001217)
44. Klein R. 2010 Scale dependent models for atmospheric flows. *Annu. Rev. Fluid Mech.* **42**, 249–274. (doi:10.1146/annurev-fluid-121108-145537)
45. Julien K, Knobloch E. 2007 Reduced models for fluid flows with strong constraints. *J. Math. Phys.* **48**, 065405. (doi:10.1063/1.2741042)
46. Hall P, Smith FT. 1991 On strongly nonlinear vortex/wave in boundary layer transition. *J. Fluid Mech.* **227**, 641–666. (doi:10.1017/S0022112091000289)
47. Kawahara G, Uhlmann M, van Veen L. 2012 The significance of simple invariant solutions in turbulent flows. *Annu. Rev. Fluid Mech.* **44**, 203–225. (doi:10.1146/annurev-fluid-120710-101228)
48. Gibson JF, Halcrow J, Cvitanović P. 2008 Visualizing the geometry of state space in plane Couette flow. *J. Fluid Mech.* **611**, 107–130. (doi:10.1017/S002211200800267X)
49. Zammert S, Eckhardt B. 2014 Periodically bursting edge states in plane Poiseuille flow. *Fluid Dyn. Res.* **46**, 041419. (doi:10.1088/0169-5983/46/4/041419)
50. Gibson JF, Brand E. 2014. Spanwise localized solutions of planar shear flows. *J. Fluid Mech.* **745**, 25–61. (doi:10.1017/jfm.2014.89)
51. Kreilos T, Gibson JF, Schneider TM. 2016 Localized travelling waves in the asymptotic suction boundary layer. *J. Fluid Mech.* **795**, R3. (doi:10.1017/jfm.2016.242)
52. Klewicki J. 2013 A description of turbulent wall flow vorticity consistent with mean dynamics. *J. Fluid Mech.* **737**, 176–204. (doi:10.1017/jfm.2013.565)