LOCALIZATION IN TRANSITIONAL SHEAR FLOWS

Tobias M. Schneider[1,2,a)], John F. Gibson[3], John Burke[3], Evan Brand[4] & John Platt[1]
[1] *School of Engineering and Applied Science, Harvard University, Cambridge, MA 02138, USA*
[2] *Max Planck Institute for Dynamics and Self-Organization, D-37073 Göttingen, Germany*
[3] *Dept. Mathematics and Statistics, University of New Hampshire, Durham, NH 03824, USA*
[4] *Dept. Mathematics and Statistics, Boston University, Boston, MA 02215, USA*

Summary Locally perturbing a parallel shear flow induces a spatially localized patch of turbulence that slowly invades the surrounding laminar flow. Though spatio-temporal patterns such as these 'turbulent spots' play a crucial role in transitional flows, the mechanism which gives rise to the observed localized structures is not well understood. We will present evidence that a well-developed theory of a localized patterns in simpler PDE models carries over to Navier-Stokes flow and allows to capture the observed patterns. Specifically, we will demonstrate the existence of multiple families of exact equilibrium and traveling wave solutions to the full Navier-Stokes equations. Those solutions share the topology of periodic solutions previously shown to play key roles in the transition to turbulence and turbulent dynamics itself but are localized in space. These localized solutions are a step towards extending the emerging dynamical systems view of transitional turbulence to spatially extended flows and uncovering the mechanisms underlying spatio-temporal turbulent patterns.

Introduction and Background

In the past decade ideas from nonlinear dynamics in combination with advances in numerical simulation techniques have laid the foundation for a new approach to study turbulence. A connection between dynamical systems and turbulence has been the subject of conjecture since the 1940s [1] .Only recently, however, has concrete progress allowed dynamical systems to be truly established as a new paradigm to study turbulence. This progress is based on the discovery of exact equilibrium and traveling-wave solutions to the full nonlinear Navier-Stokes equations. These *exact solutions*, together with their entangled stable and unstable manifolds, form a dynamical network that supports chaotic dynamics, so that turbulence can be understood as a walk among unstable solutions [2, 3]. Moreover, specific exact solutions are found to be *edge states* [4], that is, attracting objects in the stability boundary between laminar and turbulent dynamics. Thus, exact solutions play a key role both in supporting turbulence and in guiding transition.

Turbulence often does not fill the whole available domain: Instead, it coexists with laminar flow, giving rise to localized laminar-turbulent patterns. Despite its success in other areas, the emerging view of turbulence as a dynamical system has not yet been able to address the full spatio-temporal dynamics of turbulent flows. One major limitation is that exact solutions have mostly been studied in small computational domains with periodic boundary conditions. The small periodic solutions cannot capture localized spatial structures such as turbulent spots in plane Couette flow that are triggered by a localized perturbation and then grow by invading the surrounding laminar flow. Generalizing the dynamical systems picture of turbulence to the spatio-temporal aspects of flows in extended domains thus requires the existence of *localized* exact solutions.

Results: Localized invariant solutions for plane Couette flow

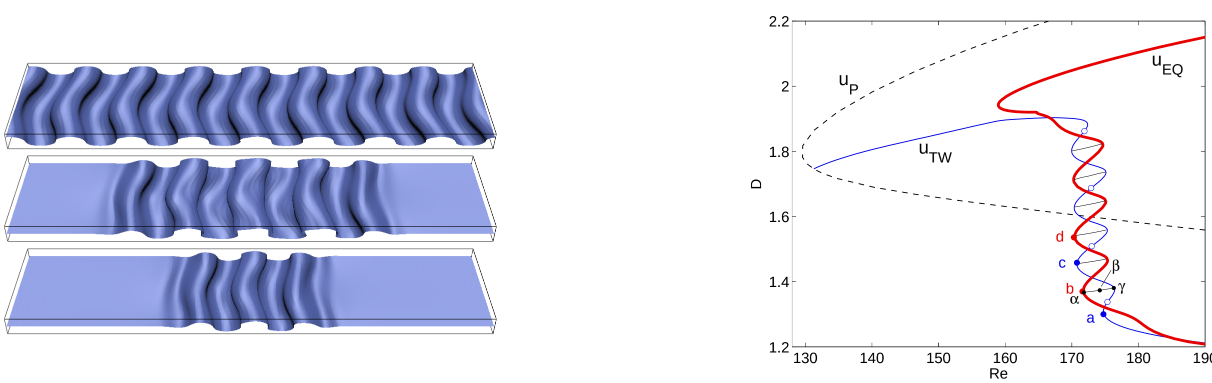

Figure 1. Spatially localized counterparts (bottom) of a periodic (top) exact invariant solution of plane Couette flow together with the full bifurcation diagram (right), where drag is plotted vs. Reynolds number: The localized solutions $(\mathrm{u}_{\mathrm{EQ}}, \mathrm{u}_{\mathrm{TW}})$ form a snakes-and-ladders structure as in the Swift-Hohenberg equation, and reconnect to the spatially periodic solution u_{P}. All branches correspond to full 3D solutions of the nonlinear Navier-Stokes equations and are computed using Newton-Krylov based continuation methods [6]. For details see [7].

a) Corresponding author. E-mail: tschneid@seas.harvard.edu

Such spatially localized exact solutions were recently constructed for for plane Couette flow, the flow between two parallel plates moving in opposite direction. These solutions are localized versions of the spatially periodic Nagata equilibrium [5]. They share the topology of periodic solutions previously shown to play key roles in the transition to turbulence and are related to their periodic counterparts by *homoclinic snaking* – a pattern-forming bifurcation scenario well studied for simpler PDE models such as the Swift-Hohenberg equation (cf. Fig. 1 and [7]).

As a function of downstream wavelength, the solutions exist for a large range of Reynolds numbers covering the relevant transitional regime in which localized turbulence is observed. Features of the bifurcation diagram vary with downstream wavelength and start to deviate from the pure Swift-Hohenberg scenario. Those variations can be related to physical properties of the flow fields, specifically to the spatial distribution of stresses exerted by the walls and an apparent 'elastic' response of the localized solution to the external forces.

In addition to the solution families related to the periodic Nagata equilibrium localized counterparts of other known periodic solutions exist. Not all of these solution families exhibit homoclinic snaking but they may also be located on isolas are show a more complex bifurcation structure. The spatially localized flow fields resemble vortical structures known to dominate the wall-near region of planar shear flows suggesting their importance for transitional turbulent dynamics.

Conclusions

All localized invariant solutions are weakly unstable with a low-dimensional unstable manifold, cover a wide range of wavelength and Reynolds number, and exists in various sizes and forms. Thus, together they may form the dynamical skeleton supporting spatio-temporally evolving turbulence and are thus a step towards understanding the mechanisms underlying spatial patterns and spatiotemporal intermittency in transitional turbulence. Further steps towards understanding laminar-turbulent patterns will include exploiting the striking similarity between the bifurcation structure of localized plane Couette solutions and well-understood patterns in simpler PDE models to derive effective amplitude equations for the turbulence intensity. Moreover, we will generalize the studies to different flow geometries including pipe but also Taylor-Couette and boundary layer flow. These investigations promise to open new avenues for understanding general spatio-temporal features of turbulence. In 1986 Pomeau envisioned to describe transitional turbulence as a nucleation phenomenon in some non-equilibrium generalization of equilibrium phase-separation concepts [8]. Our work will ideally provide a mechanistic foundation for this approach.

References

[1] Hopf E.: A mathematical example displaying features of turbulence. *Comm. Pure Appl. Math.*, **1**:303, 1948.
[2] Landford O. E.: The strange attractor theory of turbulence. *Annu. Rev. Fluid Mech.*, **14**:347–364, 1982.
[3] Gibson J. F., Halcrow J.: Visualizing the geometry of state space in plane Couette flow. *J. Fluid Mech.*, **611**:107–130, 2008.
[4] Schneider T.M., Gibson J. F., Lagha M., DeLillo F., Eckhardt B.: Laminar-turbulent boundary in plane Couette flow. *Phys. Rev. E*, **78**:037301, 2008.
[5] Nagata M.: Three-dimensional finite-amplitude solutions in plane Couette flow: bifurcation from infinity. *J. Fluid Mech.*, **217**:519–527, 1990.
[6] Gibson J. F.: www.channelflow.org.
[7] Schneider T.M., Gibson J. F., Burke J.: Snakes and ladders: localized solutions of plane Couette flow. *Phys. Rev. Lett.*, **104**:104501, 2010..
[8] Pomeau Y.: Front Motion, Metastability and Subcritical Bifurcations in Hydrodynamics. *Physica D*, **23**:3–11, 1986