she says I'm a brute, I meant to 'not attempted by the method of slices' as outlined in the slice paper. Sorry. Rewritten now.

2011-01-13 SF For the nearby trajectories the numerator is nonzero, but along a trajectory passing through a singularity doesn't the numerator approach zero θ approach the angle required to rotate the velocity into the slice? Hence the infinitely thin delta function. **ES** I am also confused about what happens in the last two paragraphs, but I'll let you two sort it out first.

2011-01-14 PC I have discussed this with Stefan, and I think he was confusing method of moving frames (where you have a freedom to rotate a (and $v(a$ with it) with method of slices, where symmetry reduction has already taken place, y cannot be rotated, and $v(y)$ is the *full* state space velocity which points wherever it pleases, even though it is evaluated on a slice point y, so $\langle v(y)|\, t'\rangle$ generically does not vanish.

2011-01-15 PC In the slice paper I make a bold claim: "While all group orbits of a generic trajectory cross the slice, the trajectory has vanishing probability to cross the lower-dimensional singular set - that is why we had to 'engineer' the slice in [our complex Lorenz equations example]. However, an ergodic trajectory might come arbitrarily close to S arbitrarily often."

This hopefully resolves a long-standing conundrum for Evangelos and me. Please CORRECT ME if I am wrong, I'm on thin ice here!

2011-01-17 Ashley The draft reads pretty well to me. There were a couple of bits I don't get but can discuss later - I've tried tessellating using all the known states in our 2.5D m=2 pipe. Intermediate success. Will post on the blog presently.

1. The use of 'template' in the abstract threw me a bit. Is there space to add ", or 'reference state'," for us dummies?
   2011-01-17 PC I like it, but where did you get the term 'reference state' from? I can find it only under computer science literature, have decided to use 'template' because (a) that's well established Marsdenite lingo [54], (b) it's very descriptive of what we actually do, and it nicely expresses my personal quest for developing a 'language of patterns,' of which plumbing is but one manifestations (electricians' cardiac dynamics labors would be another). We already accommodate the dummies the first time we define the 'template' in the body of the paper: "Think of the first pattern (represented by a point y' in the state space $\mathcal{M}$) as a 'template' [208, 54] or a 'reference state.' " In the file, 'template' is a macro, it could be easily changed.
2. Would be good to make more connections with Figure 1 in the discussion. I didn't find a reference to Fig 1(b).
   2011-01-17 PC Will do. We refer to it once: " The first equation

defines the flow confined to the slice (see Fig. 1 (b)), " but now we also refer to it in the introduction.

3. We seemed to be talking about a single reference state until section 3, so in section 2 the last paragraph on p3 about the span of several tangent vectors confused me a bit - I thought we were talking about the new method before we really were. → defer to start section 3(?)
2011-01-17 PC cannot find the text you are referring to. In any case, the discussion is for any group G, so the group tangent space for a *single slice* is N-dimensional.

4. Because of the last point, at the top of p5 when we 'pick the closest' I wasn't sure if you we already including the closest to any reference state already, but it looks like just the closest minimum from for one state. The Newton method reducing the work at this point doesn't seem entirely satisfactory, and it would not detect new minima arising and possibly getting closer later on (which would also cause a jump).
2011-01-17 PC You are right, I have now added a paragraph discussing the point you rise here.

5. In section 4 it wasn't obvious to me what was meant by *linear slices*.
2011-01-17 PC You are right: all our slices are linear. Now removed/edited.

6. Minor style difference (UK-USA difference?) - I use commas only in lists of nouns, and for other lists hyphenate where necessary e.g. "spatially-extended turbulent flows". I've only mentioned it as it occurs in the first sentence of the Intro.
2011-01-17 PC I now made it 'spatially-extended' throughout, no doubt introducing new stylistic boo-boos. Or boo boos.

7. An open issue that came up for me (can repost on blog), and appears to be alluded to at the end of section 4, is how to pick the shift of the reference states. Any ideas?
2011-01-17 PC You are right, this is THE question. The first one is for free, but that, I believe, fixes then all relative phases to the succeeding templates. It is more than alluded to: " There is a rub, though - you need to pick the phases of neighboring templates in such way that you minimize the distance from one to the next as the ant crosses the ridge. This a reflection of the flaw inherent in use of a linear, hyperplane slice globally: a slice is derived from the Euclidean notion of distance, but for nonlinear flows the distance has to be measured curvilinearly, along unstable manifolds [3, 209]. We nevertheless have to stick with tessellation by linearized tangent spaces, as curvilinear charts seem computationally too prohibitive. The *relative phase* between two different relative equilibria can be fixed, as proposed in ref. [83], by the shortest heteroclinic connection, a rigid bridge from one neighborhood to the next. "

> In the abstract of ref. [83] we say "We show, on the example of a particular small-cell Kuramoto-Sivashinsky system, how the geometry of its dynamical state space is organized by a rigid 'cage' built by heteroclinic connections between equilibria, [...]" and then in the body of the paper:
>
> " The main results presented here are: [...] (b) Existence of a rigid 'cage' built by heteroclinic connections between equilibria. "
>
> " Here we demonstrate that, for relative periodic orbits visiting the neighborhood of equilibria, if one picks any particular solution, the universe of all other solutions is rigidly fixed through a web of heteroclinic connections between them. This insight garnered from study of a 1-dimensional Kuramoto-Sivashinsky PDE is more remarkable still when applied to the plane Couette flow [210], with 3-d velocity fields and two translational symmetries. "
>
> Now, whether we have really demonstrated it is in the eye of beholder...

2011-01-17 Ashley How should I pick the shift of the reference states? Should I repeat adding, say, 100 randomly chosen turbulent states as reference states?

2011-01-17 PC No - as few slices as possible. We just need to make sure that the ridges between them are sufficiently close to each the templates template so that the inflection hyperplanes are excluded. Once templates are picked, the rest is geometry of hyperplanes (NOTHING to do with dynamics, only with the group theory) so I think checking whether the inflection hyperplane is on the far side of the tile edge (ridge between two slices) is a linear computation, to be undertaken independently of dynamics. I hope...

2011-01-17 PC Ashley made me do it: I Googled [symmetry reduction "reference state"]. There is whole new infinite regressions chain of literature in computer science [211].